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Super convergent line series' approach in solving linear programming problems

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ABSTRACT

The solutions of linear programming problems using super convergent line series' approach is considered in this paper. It is found that the results compare reasonably well with values obtained using simplex method. An algorithm known as line search exchange algorithm is used.

INTRODUCTION

Linear programming (LP) problems belong to a class of constrained convex optimization problems which have been widely discussed by several authors; see, for example, Gass (1958), Dantzig (1963) and Philip, Walter et al.(1981). The commonly used algorithms for solving linear programming problems are: the simplex method which require the use of artificial variables, and the use of surplus or slack variables, active set methods which also requires the use of artificial constraints and variables.

Umoren (1996) developed a new approach (Line search techniques which are useful in optimal experimental design) that avoids the use of slack or surplus variables. In this work, line search techniques via super convergent line series have been made use of to solve linear programming problems. The theoretical framework of super convergent line series, segmentation of response surface as well as illustrative examples are presented in subsequent sections.

THEORETICAL FRAMEWORK

In super convergent line series, each move is governed by the line equation

$$\underline{x} = \underline{x} - \rho\ \underline{d} \tag{1}$$

where $\underline{x}$ is a vector of the starting point, $\underline{d}$ is the direction vector, and ρ is the step-length. Super convergent line series, which belongs to the class of line search exchange algorithms, is defined by the following sequence of steps:

(a) Select N_k support points from the kth segment, hence make up an N-point design.

$$\zeta_N = \begin{Bmatrix} x_1, x_2, \ldots, x_n .. x_N \\ W_1, w_2, \ldots W_n, \ldots W_n \end{Bmatrix} \tag{2}$$

$$N - \sum_{k-1}^{n} Nk$$

(b) Compute the vectors $\underline{x}$, $\underline{d}$ and the step-length ρ

(c) Move to the point $\underline{x}$ = x - ρ$\underline{d}$

(d) Is $\underline{x} = x_f$ {where x_f is the minimize of f (.) } yes, Stop, No, replace $\underline{x}_m$ in ζ_N with $\underline{x}$ and thus define a new measure

$$\zeta_N = \begin{Bmatrix} x_1, x_2, \ldots, x \ldots x_N \\ w_1, w_2, \ldots w, \ldots w_N \end{Bmatrix} \tag{3}$$

ℓ Is $N_{kzN+1} \vee_k = 1, \ldots\ldots, _s$?

Yes: go to step (b) above.

No: adjust segment boundaries or take extra support points so that $N_k \geq n + 1 \vee k$ and go to step (b) above.

In the algorithm above, $\underline{x}_m$ is the point of maximum variance i.e.

$$\underline{X}^1_m M^{-1} (\zeta_N) \underline{x}_m = \max \underline{x}^1 M^{-1} (\zeta_N) \underline{x} \tag{4}$$

where $M^{-1} (\zeta_N)$ is the inverse information Matrix of ζ_N, Onukogu (1997).

SEGMENTATION OF RESPONSE SURFACE

The surface *x* is partitioned into s non-overlapping segment and we let the design measure in the sth segment be defined by

$$\zeta_s = \begin{Bmatrix} x_1, x_2, \ldots X_{NS} \\ w_1, w_2, \ldots, w_{NS} \end{Bmatrix} \tag{5}$$

$$N = \sum_{k-1}^{k} Ns$$

$w_1 = r_i$ and $\sum_{n}^{n} r_i = Ns$

The regression mode is

$$Y_s (\bullet) = c\ s\ \underline{x}_s + \underline{e}, (\bullet) \tag{6}$$

where $\underline{c}$ s (C_{0s}, C_{1s}, …., C_{ns})

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Is an n + 1 – component vector of parameters, $\underline{K}_s$ is a point in ζ_{NS}, $\underline{e}_s$ (•) is a vector of random errors. We have that in the segments, we define the direction vector by

$$\underline{do} = \sum_{s=1}^{k} \theta \ \underline{c}_s ; \sum_{s=1}^{k} \theta \quad = \quad 1, S \ \geq 0.$$

Some of the illustrative ways of partitioning

$\tilde{x} = \{X_1 \ \ X_2 ; -1 \leq \ X_1 \ \ X_2 ; -1 \leq \}$ into segments are shown in Chigbu & Ugbe (2002).

ILLUSTRATIVE EXAMPLES

Example 1 [Taha (1987), Chapter 7, problem 7-2b, pg. 296]

Minimize $Z = 5X_1 + 6X_2$

Such that $X_1 + X_2 \geq 2$

$4X_1 + X_2 \geq 4$

$X_1, X_2 \geq 0$

Solution

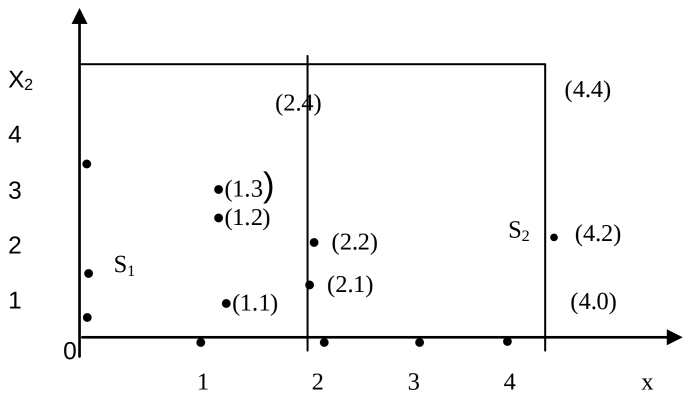

Fig.1 . Solution to the equation showing points

Referring to the diagram above, we define the segments as follows:

$S_1 = \{x_1, x_2, 0 \leq x_1 \leq 2, 0 \leq x_2 \leq 4\}$, $S_2 = \{x_1, x_2; 2 \leq x_1 \leq 4, 0 \leq x_2 \leq 2\}$

The design matrices are:

$$x_1 = \begin{bmatrix} 0 & 4 \\ 2 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \text{ and } x_2 = \begin{bmatrix} 1 & 1 \\ 4 & 0 \\ 2 & 1 \\ 2 & 2 \end{bmatrix}$$

The inverses of the information matrices are:

$(X_1 \quad X_1)^{-1} = \begin{pmatrix} 0.117.6 & SYM \\ -0.0196 & 0.0588 \end{pmatrix}$ and $(X_2 \quad X_2)^{-1} = \begin{pmatrix} 0.0577 & SYM \\ -0.0796 & 0.2692 \end{pmatrix}$

Where 'SYM' means that $(X_1X_1)^{-1}$ and $(X_2 X_2)^{-1}$ are symmetric matrices.

The matrices of coefficients of convex combination of the inverses of the information matrices are:

H_1 = diag {0.3298, 0.8207} and H_2 = diag {0.8215, 0.2134}

These matrices are normalized to give

H_1^* = diag {0.4032, 0.9769} and H_2^* = diag {0.8215, 0.2134}.

The direction vector, $\underline{d} = M^{-1} (\varsigma\ N)\underline{Z} = \begin{pmatrix} 5.0063 \\ 6.0015 \end{pmatrix}$,

this is normalized to give

$\underline{d}^* = \begin{pmatrix} 0.6405 \\ 0.7680 \end{pmatrix}, X = \Sigma\, wi\ xi \quad = \begin{pmatrix} 1.6877 \\ 1.0001 \end{pmatrix}$

The step – length, p* = 0.4883

$\underline{X}^* = X^* - p^* \underline{d}^* = \begin{pmatrix} 1.4 \\ 0.6 \end{pmatrix}$

Therefore, Min = 10.6

This value is close to the optimum value got by Taha (1987) using simplex method, as Min = 10.0

Example 2 [Hillier and Lieberman (1995), chapter 4, pg. 82]

Maximize Z = $3x_1 + 5x_2$

Such that $X_1, \leq 4$

$2x_2 \leq 12$

$2x_1 + 3x_2 \leq 18$

$X_1, x_2 \geq 0$

These matrices are normalized to give

H_1^* = diag (0.1863, 0.9264) and H_2^* = diag (0.9825, 0.3765)

The direction vector $\underline{d} = M^{-1} (\varsigma\ N)\underline{Z} \quad = \begin{bmatrix} 3.0072 \\ 5.0047 \end{bmatrix}$, which is normalized to give

$\underline{d}^* = \begin{bmatrix} 0.5150 \\ 0.8572 \end{bmatrix}, \cdots \underline{X}^* = \Sigma wixi \quad = \begin{bmatrix} 2.2927 \\ 1.9921 \end{bmatrix}$,

the step – length, p* = -3.351

$\underline{x}^* = x^* - p^* \underline{d}^* = \begin{bmatrix} 4.0 \\ 4.8 \end{bmatrix}$ therefore, max. z = 36,

which is exactly the same as the optimal value obtained by Hillier & Lieberman (1995), by the simplex method as max. Z = 36.

CONCLUSION

In conclusion,the solutions obtained for the two illustrative examples compared reasonably with those obtained by the aforementioned authors using the simplex method. The examples used were, however, chosen because we could easily partition the response surface into segments (as shown in figures 1) and then select support points.

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